FYJC - MATHEMATICS & STATISTICS

PAPER - II

COMBINATIONS

COMBINATION - QSET1

SUMS ON SELECTION OF COMMITTEE / TEAM

01. there are seven men and three women . Find the number of ways in which a committee of 6 can be formed from these if the committee is to include at least two women **ans : 140**

02. From amongst 8 men and 5 women , a committee of 5 is to be formed so as to include at least 3 women . find the number of ways in which this can be done **ans : 321**

03. a committee of 5 is to be formed out of 6 gents and 4 ladies . In how many ways can this be done if at most two ladies are included **ans : 186**

04. Out of 6 teachers and four boys , a committee of eight is to be formed . In how many ways can this be done when there should not be less than 4 teachers in the committee **ans : 45**

05. from 4 accountants , 3 lawyers and 5 salesmen , a committee of 7 is to be formed . In how many ways can this be done if it contains at least 4 salesmen **ans : 196**

06. a question paper consists of 11 questions divided into two sections I and II . Section I consists of 5 questions and section II consists of 6 questions . In how many ways can a student select 6 questions taking at least 2 questions from each section **ans : 425**

07. a cricket team of 11 players is to be selected from a group of 15 players out of whom there are 6 are bowlers and 3 are wicket keepers . The team should contain exactly 1 wicket keeper and at least 4 bowlers . Find the number of ways in which this can be done **ans : 198**

08. A committee of 12 persons is to be formed from 9 women and 8 men . In how many ways can this be done if men are in majority **ans : 1134**

COMBINATION - QSET2

SUMS ON INCLUDE / EXCLUDE

01. In how many ways can 5 students be selected out of 11 students if

a) 2 particular students are included **ans : 84**

b) 2 particular students are not included **ans : 126**

02. there are 15 players including A , B & C . Find the number of ways in which cricket team of 11 can be chosen if

a) A is already selected as captain **ans:1001**

b) B is injured & is not available **ans : 364**

c) A is selected as captain & at the same time B is not available **ans : 286**

03. The staff of the bank consists of the manager , the deputy manager and 10 other officers . A committee of 4 is to be selected . Find the number of ways in which this can be done so as to

a) include the manager **ans : 165**

b) include the manger but not the deputy manager **ans : 120**

c) neither the manager nor the deputy manager **ans : 210**

04. Out of 4 officers and 10 clerks in an office , a committee consisting of 2 officers and 3 clerks is to be formed . In how many ways can this be done if

a) one particular clerk must be on the committee **ans : 216**

b) one particular officer cannot be on the committee **ans : 360**

05. A student is to answer eight out of 10 questions in an examination

a) how many choices has he if he must answer the first three questions **ans : 21**

b) how many choices has he if he must answer at least four out of first five **ans : 35**

06. in how many ways can 18 objects be divided into 3 groups containing 9 , 6 & 3 objects respectively **ans : $^{18}C_9 \times {}^{9}C_6 \times {}^{3}C_3$**

07. in how many ways can 15 things be divided into 3 groups containing 8 , 4 and 3 things respectively **ans : $^{15}C_8 \times {}^{7}C_4 \times {}^{3}C_3$**

08. from a class of 25 students 10 are to be chosen for a project work . There are 3 students who decide that either all of them will join or none will join . In how many ways can they be chosen . **ans : $^{22}C_{10} + {}^{22}C_7$**

09. a boy has 3 library tickets and 8 books of his interest in the library . Of these 8 books , he does not want to borrow Chemistry part II , unless Chemistry part I is borrowed . In how many ways can he choose three books to be borrowed . **ans : 41**

10. In how many ways can a committee of 3 ladies and 4 gents be chosen from 8 ladies and 7 gents . What is the number of ways if Miss X refuses if Mr Y is a member . **ans: 1540**

COMBINATION - QSET3

SUMS ON CHORDS – LINES – TRIANGLES – POLYGONS

01. How many chords can be drawn through 21 points on a circle **ans : 210**

02. Find maximum number of diagonals that can be drawn in n – sided polygon where

1) n = 12 2) n = 15 3) decagon **ans: 54 ; 90 ; 35**

03. Find the number of straight lines obtained by joining 10 points on a plane , if

a) no three points are collinear **ans : 45**

b) four points are collinear **ans : 40**

04. there are 15 points in a plane out of which 5 are collinear . Prove that we can obtain 96 straight lines by joining these points in pairs .

05. there are 22 points in a plane of which p points are collinear . If 211 different lines can be obtained by joining them find p **ans : 7**

06. Find the number of triangles obtained by joining 10 points on a plane , if

a) no three of them are collinear **ans : 120**

b) four points are collinear **ans : 116**

07. there are 15 points in a plane out of which 5 are collinear . Prove that there are 445 triangles with vertices at these points

08. If there are 12 points in a plane out of which 'p' points are collinear , find the value of 'p' for which 185 triangles can be obtained by joining these 12 points . **ans : 7**

09. Each of a set of 5 parallel lines cuts each one of another set of 4 parallel lines . How many different parallelograms can be formed **ans : 60**

10. at the end of meeting , everyone had shaken hands with every one else . It was found that 45 handshakes were exchanged . How many members were present at the meeting . **ans : 10**

COMBINATION - QSET4

SUMS ON $^nP_r = {^nC_r} \cdot r!$; $^nC_r = \dfrac{n!}{r!(n-r)!}$

01. $^nC_4 = 5\ {^nP_3}$, find n **ans :123**

02. $^nP_r = 120$ & $^nC_r = 20$, find n and r **ans :6,3**

03. $^nP_r = 720$ & $^nC_r = 120$, find n and r **ans :10,3**

04. $^nP_r = 32760$ & $^nC_r = 1365$, find n and r **ans :4,15**

05. $^nC_6 : {^{n-3}C_3} = 33 : 4$, find n **ans : 11**

06. $^{14}C_{2r} : {^{10}C_{2r-4}} = 143 : 10$, find r **ans : 4**

07. $^{28}C_{2r} : {^{24}C_{2r-4}} = 225 : 11$, find r **ans : 7**

08. $^{10}C_{r+2} : {^{10}C_r} = 10 : 21$, find r **ans : 5**

09. $^nC_{r-1} : {^nC_r} : {^nC_{r+1}} = 20 : 35 : 42$, find n & r **ans: 10,4**

10. $^nC_{r-1} = 495$; $^nC_r = 220$; $^nC_{r+1} = 66$, find n & r **ans : 12,9**

$$\boxed{{^nC_r} + {^nC_{r-1}} = {^{n+1}C_r}}$$

11. $^{14}C_5 + {^{14}C_6} + {^{15}C_7} + {^{16}C_8} = {^{17}C_x}$, find x **ans : 8,9**

12. $^{25}C_4 + {^{25}C_5} + {^{26}C_6} + {^{27}C_7} = {^{28}C_r}$, find r **ans :7,21**

13. $^{12}C_5 + 2.\ {^{12}C_4} + {^{12}C_3} = {^{14}C_x}$, find x **ans : 5,9**

14. $^{47}C_4 + \sum_{r=1}^{5} {^{52-r}C_3}$. **ans : $^{52}C_4$**

15. $^{20}C_{2n} = {^{20}C_{n^2-4}}$ find n

13. $^{18}C_{2r} = {^{18}C_{r^2+3}}$ find r

COMBINATION - SOLUTION TO QSET-1

01. there are seven men and three women . Find the number of ways in which a committee of 6 can be formed from these if the committee is to include at least two women

7 men , 3 women

committee of 6 (at least 2 women)

Case 1 : Committee contains 4 men & 2 women

This can formed in $= {}^7C_4 \times {}^3C_2 . = {}^7C_3 \times {}^3C_1$

$= 35 \times 3 = 105$ ways

Case 2 : Committee contains 3 men & 3 women

This can formed in $= {}^7C_3 \times {}^3C_3 . = 35 \times 1 = 35$ ways

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By fundamental principle of ADDITION

Total ways of forming the committee = 140

02. From amongst 8 men and 5 women , a committee of 5 is to be formed so as to include at least 3 women . find the number of ways in which this can be done

8 men , 5 women

committee of 5 (at least 3 women)

Case 1 : Committee contains 2 men & 3 women

This can formed in $= {}^8C_2 \times {}^5C_3 . = {}^8C_2 \times {}^5C_2 .$

$= 28 \times 10 = 280$ ways

Case 2 : Committee contains 1 men & 4 women

This can formed in $= {}^8C_1 \times {}^5C_4 . = {}^8C_1 \times {}^5C_1 .$

$= 8 \times 5 = 40$ ways

Case 3 : Committee contains no man & 5 women

This can formed in $= {}^5C_5 .$ = 1 way

By fundamental principle of ADDITION

Total ways of forming the committee = 321

03. a committee of 5 is to be formed out of 6 gents and 4 ladies . In how many ways can this be done if at most two ladies are included

6 gents and 4 ladies

committee of 5 (at most two ladies)

Case 1 : Committee contains 5 gents & no ladies

This can formed in $= {}^6C_5 \times {}^4C_0 . = {}^6C_1 \times {}^4C_0 .$

$= 6 \times 1 = 6$ ways

Case 2 : Committee contains 4 gents & 1 lady

This can formed in $= {}^6C_4 \times {}^4C_1 . = {}^6C_2 \times {}^4C_1 .$

$= 15 \times 4 = 60$ ways

Case 3 : Committee contains 3 gents & 2 ladies

This can formed in $= {}^6C_3 \times {}^4C_2 . = 20 \times 6 = 120$ ways

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By fundamental principle of ADDITION

Total ways of forming the committee = 186

04. Out of 6 teachers and four boys , a committee of eight is to be formed . In how many ways can this be done when there should not be less than 4 teachers in the committee

6 teachers , 4 boys

committee of 8 (not less than 4 teachers)

Case 1 : Committee contains <u>4 teachers</u> & 4 boys

This can formed in $= {}^6C_4 \times {}^4C_4 . = {}^6C_2 \times {}^4C_4 .$

$= 15 \times 1 = 15$ ways

Case 2 : Committee contains <u>5 teachers</u> & 3 boys

This can formed in $= {}^6C_5 \times {}^4C_3 . = {}^6C_1 \times {}^4C_1 .$

$= 6 \times 4 = 24$ ways

Case 3 : Committee contains <u>6 teachers</u> & 2 boys

This can formed in $= {}^6C_6 \times {}^4C_2 . = 1 \times 6 = 6$ ways

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By fundamental principle of ADDITION

Total ways of forming the committee = 45

05. from 4 accountants , 3 lawyers and 5 salesmen , a committee of 7 is to be formed . In how many ways can this be done if it contains at least 4 salesmen **ans : 196**

4 accountants , 3 lawyers and 5 salesmen

committee of 7 (at least 4 salesmen)

Case 1 : Committee contains <u>4 salesmen</u> & 3 others

This can formed in $= {}^5C_4 \times {}^7C_3 . = {}^5C_1 \times {}^7C_3$

$= 5 \times 35 = 175$ ways

Case 2 : Committee contains <u>5 salesmen</u> & 2 others

This can formed in $= {}^5C_5 \times {}^7C_2 . = 1 \times 21 = 21$ ways

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By fundamental principle of ADDITION

Total ways of forming the committee = 196

06. a question paper consists of 11 questions divided into two sections I and II . Section I consists of 5 questions and section II consists of 6 questions . In how many ways can a student select 6 questions taking at least 2 questions from each section

Section I consists of 5 questions and section II consists of 6 questions
student select 6 questions taking at least 2 questions from each section

Case 1 : students selects 2 Q's from Section I & 4 Q's from Section II
This can done in $= {}^5C_2 \times {}^6C_4 . = {}^5C_2 \times {}^6C_2 .$
$= 10 \times 15 = 150$ ways

Case 2 : students selects 3 Q's from Section I & 3 Q's from Section II
This can done in $= {}^5C_3 \times {}^6C_3 . = {}^5C_2 \times {}^6C_3 .$
$= 10 \times 20 = 200$ ways

Case 3 : students selects 4 Q's from Section I & 2 Q's from Section II
This can done in $= {}^5C_5 \times {}^6C_2 . = {}^5C_1 \times {}^6C_2 .$
$= 5 \times 15 = 75$ ways

By fundamental principle of ADDITION
Total ways = 425

07. a cricket team of 11 players is to be selected from a group of 15 players out of whom there are 6 are bowlers and 3 are wicket keepers . The team should contain exactly 1 wicket keeper and at least 4 bowlers . Find the number of ways in which this can be done

6 Bowlers , 3 wicket keepers & 6 batsmen
a cricket team of 11 players is to be selected (exactly 1 wicket keeper & at least 4 bowlers)

Case 1 : Team contains <u>4 Bowlers</u> , 1 wicket keepers & 6 batsmen
This can formed in $= {}^6C_4 \times {}^3C_1 \times {}^6C_6 . = {}^6C_2 \times {}^3C_1 \times {}^6C_6$
$= 15 \times 3 \times 1 = 45$ ways

Case 2 : Team contains <u>5 Bowlers</u> , 1 wicket keepers & 5 batsmen
This can formed in $= {}^6C_5 \times {}^3C_1 \times {}^6C_5 . = {}^6C_1 \times {}^3C_1 \times {}^6C_1$
$= 6 \times 3 \times 6 = 108$ ways

Case 3 : Team contains <u>6 Bowlers</u> , 1 wicket keepers & 4 batsmen
This can formed in $= {}^6C_6 \times {}^3C_1 \times {}^6C_4 . = {}^6C_6 \times {}^3C_1 \times {}^6C_2$
$= 1 \times 3 \times 15 = 45$ ways

By fundamental principle of ADDITION
Total ways of forming the committee = 198

08. A committee of 12 persons is to be formed from 9 women and 8 men . In how many ways can this be done if men are in majority **ans : 1134**

9 women , 8 men

committee of 12 (men are in majority)

Case 1 : Committee contains 5 women & 7 men

This can formed in $= {}^9C_5 \times {}^8C_7 = {}^9C_4 \times {}^8C_1$

$= 126 \times 8 = 1008$ ways

Case 2 : Committee contains 4 women & 8 men

This can formed in $= {}^9C_4 \times {}^8C_8 = 126 \times 1 = 126$ ways

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By fundamental principle of ADDITION

Total ways of forming the committee = 1134

COMBINATION - SOLUTION TO Q SET-2

01. In how many ways can 5 students be selected out of 11 students if

a) 2 particular students are included

Since 2 particular students are included , the remaining 3 students have to be selected from the remaining 9 students .

This can be done in $= {}^9C_3 = 84$ ways

b) 2 particular students are not included

Since 2 particular students are not included , the 5 students have to be selected from the remaining 9 students .

This can be done in $= {}^9C_5 = {}^9C_4 = 126$ ways

02. there are 15 players including A , B & C . Find the number of ways in which cricket team of 11 can be chosen if

a) A is already selected as captain

Since A is already selected as captain , remaining 10 players have to be selected from the remaining 14 players .

This can be done in $= {}^{14}C_{10} = {}^{14}C_4 = 1001$ ways

b) B is injured & is not available

Since B is injured & is not available , the 11 players have to be selected from the remaining 14 players .

This can be done in $= {}^{14}C_{11} = {}^{14}C_3 = 364$ ways

c) A is selected as captain & at the same time B is not available

Since A is selected as captain & at the same time B is not available , the remaining 10 players have to be selected from the remaining 13 players .

This can be done in $= {}^{13}C_{10} = {}^{13}C_3 = 286$ ways

03. The staff of the bank consists of the manager , the deputy manager and 10 other officers . A committee of 4 is to be selected . Find the number of ways in which this can be done so as to

a) include the manager

Since manager is included , remaining 3 members have to be selected from the remaining 11 members (1 deputy manager + 10 officers) .

This can be done in $= {}^{11}C_3 = 165$ ways

b) include the manger but not the deputy manager

since the manager is included but not the deputy manager , remaining 3 members have to be selected from the remaining 10 officers .

This can be done in $= {}^{10}C_3 = 120$ ways

c) neither the manager nor the deputy manager

since neither the manager nor the deputy manager is included , the 4 members have to be selected from the remaining 10 officers

This can be done in $= {}^{10}C_4 = 210$ ways

04. Out of 4 officers and 10 clerks in an office , a committee consisting of 2 officers and 3 clerks is to be formed . In how many ways can this be done if

a) one particular clerk must be on the committee

since one particular clerk must be on the committee , the remaining 2 clerks have to be selected from the remaining 9 clerks . This can be done in $^9C_2$ ways

Having done that ,

2 officers have to be selected from the 4 officers . This can be done in $^4C_2$ ways

By fundamental principle of Multiplication ,

Total ways of forming the committee = $^9C_2 \times {}^4C_2 = 36 \times 6 = 216$

b) one particular officer cannot be on the committee

since one particular officer must not be on the committee , the 2 officers have to be selected from the remaining 3 officers . This can be done in $^3C_2$ ways

Having done that ,

3 clerks have to be selected from 10 clerks . This can be done in $^{10}C_3$ ways

By fundamental principle of Multiplication ,

Total ways of forming the committee = $^3C_2 \times {}^{10}C_3 = 3 \times 120 = 360$

05. A student is to answer eight out of 10 questions in an examination

a) how many choices has he if he must answer the first three questions

since the student must answer first 3 questions , he has to then select the remaining 5 questions from the remaining 7 questions

This can be done in $= {}^7C_5 = {}^7C_2 = 21$ ways

b) how many choices has he if he must answer at least four out of first five questions

Case 1 : Student answers 4 Q's from first 5 Q's and 4 Q's from next 5 Q's

This can be done in $= {}^5C_4 \times {}^5C_4 = {}^5C_1 \times {}^5C_1$

$= 5 \times 5 = 25$ ways

Case 2 : Student answers 5 Q's from first 5 Q's and 3 Q's from next 5 Q's

This can be done in $= {}^5C_5 \times {}^5C_3 = {}^5C_5 \times {}^5C_2$

$= 1 \times 10 = 10$ ways

By fundamental principle of addition

Total ways = 45

06. in how many ways can 18 objects be divided into 3 groups containing 9 , 6 & 3 objects respectively

First 9 objects have to be selected from the 18 objects . This can be done in ${}^{18}C_9$ways

Having done that ; next 6 objects have to be selected from remaining 9 objects . This can be done in ${}^{9}C_6$ ways

Having done that ; last 3 objects have to be selected from the remaining 3 objects . This can be done in ${}^{3}C_3$ ways . Hence total ways = ${}^{18}C_9 \times {}^{9}C_6 \times {}^{3}C_3$

07. in how many ways can 15 things be divided into 3 groups containing 8 , 4 and 3 things respectively

First 8 things have to be selected from the 15 things . This can be done in ${}^{15}C_8$ ways

Having done that ; next 4 things have to be selected from remaining 7 things . This can be done in ${}^{7}C_4$ ways

Having done that ; last 3 things have to be selected from the remaining 3 things . This can be done in ${}^{3}C_3$ ways .

Hence <u>By Fundamental Principle of Multiplication</u> : total ways = ${}^{15}C_8 \times {}^{7}C_4 \times {}^{3}C_3$

08. from a class of 25 students 10 are to be chosen for a project work . There are 3 students who decide that either all of them will join or none will join . In how many ways it can be done

Case 1 : <u>3 students decide : all 3 of them will join</u>

Since all 3 students will join , the remaining 7 students have to be selected from the remaining 22 students . This can be done in ${}^{22}C_7$ ways

Case 2 : <u>3 students decide : all 3 of them will not join</u>

Since all 3 students will join , the 10 students have to be selected from the remaining 22 students . This can be done in ${}^{22}C_{10}$ ways

<u>By fundamental principle of addition</u>

Total ways = ${}^{22}C_7 + {}^{22}C_{10}$

09. a boy has 3 library tickets and 8 books of his interest in the library . Of these 8 books , he does not want to borrow Chemistry part II , unless Chemistry part I is borrowed . In how many ways can he choose three books to be borrowed .

Case 1 : Chemistry part I is borrowed

Since Chemistry part I is already borrowed , the boy has to now select remaining 2 books from the remaining 7 books .

This can be done in $^7C_2 = 21$ ways

Case 2 : Chemistry part I is NOT borrowed

Since Chemistry part I is NOT borrowed , the boy will not borrow Chemistry part II Hence the boy has to now select 3 books from the remaining 6 books .

This can be done in $^6C_3 = 20$ ways

By Fundamental principle of Addition : Total ways = $21 + 20 = 41$

10. In how many ways can a committee of 3 ladies and 4 gents be chosen from 8 ladies and 7 gents . What is the number of ways if Miss X refuses if Mr Y is a member .

Case 1 : Mr Y is a member

Since Mr Y is a member , the remaining 3 gents have to be selected from the remaining 6 gents . This can be done in $^6C_3$ ways

Since Mr Y is a member , Miss X will not the member . Therefore the 3 ladies have to be selected from the remaining 7 ladies . This can be done in $^7C_3$ ways

By fundamental principle of Multiplication

No of ways of forming such a committee $= {^6C_3} \times {^7C_3} = 20 \times 35 = 700$

Case 2 : Mr Y is NOT a member

Since Mr Y is not a member , the 4 gents have to be selected from the remaining 6 gents . This can be done in $^6C_4$ ways

Having done that ,

the 3 ladies have to be selected from the remaining 8 ladies . This can be done in $^8C_3$ ways

By fundamental principle of Multiplication

No of ways of forming such a committee $= {^6C_4} \times {^8C_3} = {^6C_2} \times {^8C_3}$

$= 15 \times 56 = 840$

By fundamental principle of addition

Total ways of forming the committee $= 700 + 840 = 1540$

COMBINATION - SOLUTION TO Q SET-3

01. How many chords can be drawn through 21 points on a circle

2 points on a circle define a chord

$\therefore$ number of chords that can be drawn = ${}^{21}C_2$ = 210

02. Find maximum number of diagonals that can be drawn in n – sided polygon where

1) n = 12 2) n = 15 3) decagon

1) 12 – sided polygon

12 sided polygon

12 points

2 points define a line

$\therefore$ number of line that can be drawn

= ${}^{12}C_2$ = 66

But 12 are sides

$\therefore$ No . of diagonals = 66 – 12

= 54

2) 15 – sided polygon

15 sided polygon

15 points

2 points define a line

$\therefore$ number of line that can be drawn

= ${}^{15}C_2$ = 105

But 15 are sides

$\therefore$ No . of diagonals = 105 – 15

= 90

03. Find the number of straight lines obtained by joining 10 points on a plane , if

a) no three points are collinear

10 points

2 points define a line

$\therefore$ number of line that can be drawn = ${}^{10}C_2$ = 45

b) four points are collinear

10 points

2 points define a line

$\therefore$ number of line that can be drawn = ${}^{10}C_2$ = 45

But 4 points are collinear

Number of lines wrongly counted in these 4 collinear points = ${}^{4}C_2$ = 6 instead of 1

Hence

actual lines that can be drawn = 45 – 6 + 1 = 40

04. there are 15 points in a plane out of which 5 are collinear . Prove that we can obtain 96 straight lines by joining these points in pairs .

15 points

2 points define a line

$\therefore$ number of line that can be drawn $= {}^{15}C_2 = 105$

But 5 points are collinear

Number of lines wrongly counted in these 5 collinear points $= {}^{5}C_2 = 10$ instead of 1

Hence actual lines that can be drawn = $105 - 10 + 1 = 96$

05. there are 22 points in a plane of which p points are collinear . If 211 different lines can be obtained by joining them find p

22 points

2 points define a line

$\therefore$ number of line that can be drawn $= {}^{22}C_2 = 231$

But p points are collinear

Number of lines wrongly counted in these 5 collinear points $= {}^{p}C_2$ instead of 1

Hence actual lines that can be drawn

$231 - {}^{p}C_2 + 1 = 211$ …… given

$232 - {}^{p}C_2 = 211$

${}^{p}C_2 = 21$

$\frac{p(p-1)}{2} = 21$

$p(p-1) = 42$

$p(p-1) = 7.6$ $\therefore$ On Comparison $n = 7$

06. Find the number of triangles obtained by joining 10 points on a plane , if

a) no three of them are collinear

10 points

3 points define a triangle

$\therefore$ number of triangles that can be drawn $= {}^{10}C_3 = 120$

b) four points are collinear

10 points

3 points define a triangle

$\therefore$ number of line that can be drawn = ${}^{10}C_3$ = 120

But 4 points are collinear

Number of triangles wrongly counted in these 4 collinear points

= ${}^{4}C_3$ = ${}^{4}C_1$ = 4 instead of 0

Hence

actual triangles that can be drawn = 120 – 4 + 0 = 116

07. there are 15 points in a plane out of which 5 are collinear . Prove that there are 445 triangles with vertices at these points

15 points

3 points define a triangle

$\therefore$ number of line that can be drawn = ${}^{15}C_3$ = 455

But 5 points are collinear

Number of triangles wrongly counted in these 5 collinear points

= ${}^{5}C_3$ = ${}^{5}C_2$ = 10 instead of 0

Hence

actual triangles that can be drawn = 455 – 10 + 0 = 445

08. If there are 12 points in a plane out of which 'p' points are collinear , find the value of 'p' for which 185 triangles can be obtained by joining these 12 points .

12 points

3 points define a triangle

$\therefore$ number of line that can be drawn = ${}^{12}C_3$ = 220

But p points are collinear

Number of triangles wrongly counted in these 5 collinear points

= ${}^{p}C_3$ instead of 0

Hence

actual triangles that can be drawn

220 – ${}^{p}C_3$ = 185

${}^{p}C_3$ = 35

$$\frac{p(p-1)(p-2)}{3.2.1} = 35$$

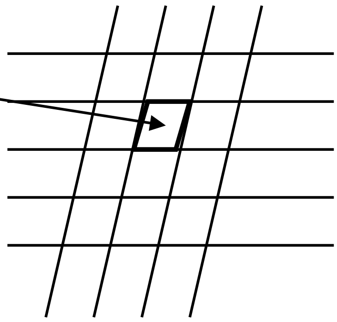
2	210
3	105
5	35
7	7
	1

$$p(p-1)(p-2) = 210$$

$$p(p-1)(p-2) = 7.6.5 \qquad \therefore \text{ On Comparison } \; n = 7$$

09. Each of a set of 5 parallel lines cuts each one of another set of 4 parallel lines . How many different parallelograms can be formed **ans : 60**

2 lines from set A and 2 lines from set B define a parallelogram therefore number of parallelograms that can be drawn



= ${}^5C_2 \times {}^4C_2$

= 10 x 6

= 60

10. at the end of meeting , everyone had shaken hands with every one else . It was found that 45 handshakes were exchanged . How many members were present at the meeting .

'n' be the number of persons in the meeting

2 persons make a handshake

$\therefore$ number of handshakes = ${}^nC_2$ = 45 ……… Given

$$\frac{n(n-1)}{2} = 45$$

$$n(n-1) = 90$$

$$n(n-1) = 10.9 \qquad \therefore \text{ On Comparison } \; n = 10$$

COMBINATION - SOLUTION TO Q SET-4

01. *${}^{n}C_{4} = 5\,{}^{n}P_{3}$, find n*

${}^{n}C_{4} = 5\,{}^{n}P_{3}$

$$\frac{\cancel{n!}}{(n-4)!\,.\,4!} = 5.\frac{\cancel{n!}}{(n-3)!}$$

$$\frac{(n-3)!}{(n-4)!} = 5.4!$$

$$\frac{(n-3)\,(n-4)!}{(n-4)!} = 5.4!$$

$(n - 3) = 5(24)$

$n - 3 = 120$

$n = 123$

02. *${}^{n}P_{r} = 120$ & ${}^{n}C_{r} = 20$, find n and r*

${}^{n}P_{r} = {}^{n}C_{r}\,.\,r!$

$120 = 20\,.\,r!$

$r! = 6$

$r! = 3!$

$r = 3$

Now ;

${}^{n}P_{r} = 120$

${}^{n}P_{3} = 120$

$$\frac{n!}{(n-3)!} = 120$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 120$$

$n(n - 1)(n - 2) = 120$

$n(n - 1)(n - 2) = 6.5.4$

On Comparing ; $n = 6$

03. *${}^{n}P_{r} = 720$ & ${}^{n}C_{r} = 20$, find n and r*

${}^{n}P_{r} = {}^{n}C_{r}\,.\,r!$

$720 = 120\,.\,r!$

$r! = 6$

$r! = 3!$

$r = 3$

Now ;

${}^{n}P_{r} = 720$

${}^{n}P_{3} = 720$

$$\frac{n!}{(n-3)!} = 720$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 720$$

$n(n - 1)(n - 2) = 720$

$n(n - 1)(n - 2) = 10.9.8$

On Comparing ; $n = 10$

05. $^{n}C_{6} : {}^{n-3}C_{3} = 33 : 4$, *find n*

$$\frac{^{n}C_{6}}{^{n-3}C_{3}} = \frac{33}{4}$$

$$\frac{\frac{n!}{(n-6)!.6!}}{\frac{(n-3)!}{(n-3-3)!.3!}} = \frac{33}{4}$$

$$\frac{\frac{n!}{\cancel{(n-6)!}.6!}}{\frac{(n-3)!}{\cancel{(n-6)!}.3!}} = \frac{33}{4}$$

$$\frac{n!}{6!} \times \frac{3!}{(n-3)!} = \frac{33}{4}$$

$$\frac{n!}{(n-3)!} \times \frac{3!}{6!} = \frac{33}{4}$$

$$\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} \times \frac{3!}{6.5.4.3!} = \frac{33}{4}$$

$$\frac{n(n-1)(n-2)}{6.5.4} = \frac{33}{4}$$

$$n(n-1)(n-2) = 33 . 6 .5$$

$$n(n-1)(n-2) = 990$$

$$n(n-1)(n-2) = 11 \times 10 \times 9$$

On Comparing ; $n = 11$

06. $^{14}C_{2r} : {}^{10}C_{2r-4} = 143 : 10$, *find r*

$$\frac{^{14}C_{2r}}{^{10}C_{2r-4}} = \frac{143}{10}$$

$$\frac{\frac{14!}{(14-2r)!\,.\,2r!}}{\frac{10!}{(10-2r+4)!\,.\,(2r-4)!}} = \frac{143}{10}$$

$$\frac{\frac{14!}{\cancel{(14-2r)!}\,.\,2r!}}{\frac{10!}{\cancel{(14-2r)!}\,.\,(2r-4)!}} = \frac{143}{10}$$

$$\frac{14!}{2r!} \times \frac{(2r-4)!}{10!} = \frac{143}{10}$$

$$\frac{(2r-4)!}{2r!} \times \frac{14!}{10!} = \frac{143}{10}$$

$$\frac{\cancel{(2r-4)!}}{2r\,.(2r-1)(2r-2)(2r-3)\cancel{(2r-4)!}} \times \frac{14.13.12.11.\cancel{10!}}{\cancel{10!}} = \frac{143}{10}$$

$$\frac{14.\cancel{13}.12.\cancel{11}}{2r\,.(2r-1)(2r-2)(2r-3)} = \frac{\cancel{143}}{10}$$

$$2r\,.(2r-1)(2r-2)(2r-3) = 14 . 12 . 10$$

$$2r\,.(2r-1)(2r-2)(2r-3) = 7.2 . 6.2 . 5.2$$

$$2r\,.(2r-1)(2r-2)(2r-3) = 8 . 7 . 6 . 5$$

On comparing ; $2r = 8 \quad \therefore \ r = 4$

07. $^{28}C_{2r} : {}^{24}C_{2r-4} = 225 : 11$, *find r*

$$\frac{^{28}C_{2r}}{^{24}C_{2r-4}} = \frac{225}{11}$$

$$\frac{\dfrac{28!}{(28-2r)!\,.\,2r!}}{\dfrac{24!}{(24-2r+4)!\,.\,(2r-4)!}} = \frac{225}{11}$$

$$\frac{\dfrac{28!}{\cancel{(28-2r)!}\,.\,2r!}}{\dfrac{24!}{\cancel{(28-2r)!}\,.\,(2r-4)!}} = \frac{225}{11}$$

$$\frac{28!}{2r!} \times \frac{(2r-4)!}{24!} = \frac{225}{11}$$

$$\frac{(2r-4)!}{2r!} \times \frac{28!}{24!} = \frac{225}{11}$$

$$\frac{\cancel{(2r-4)!}}{2r\,.(2r-1)(2r-2)(2r-3)\cancel{(2r-4)!}} \times \frac{28.27.26.25.\cancel{24!}}{\cancel{24!}} = \frac{225}{11}$$

$$\frac{28.\overset{3}{\cancel{27}}.26.\cancel{25}}{2r\,.(2r-1)(2r-2)(2r-3)} = \frac{\cancel{225}^{\,9}}{11}$$

$$2r\,.(2r-1)(2r-2)(2r-3) = 28\,.\,3\,.\,26\,.\,11$$

$$2r\,.(2r-1)(2r-2)(2r-3) = 14.2.3.13.2.11$$

$$2r\,.(2r-1)(2r-2)(2r-3) = 14.13.12.11$$

On comparing ; $2r = 14 \quad \therefore\ r = 7$

08. $^{10}C_{r+2} : {}^{10}C_r = 10 : 21$, *find r*

$$\frac{^{10}C_{r+2}}{^{10}C_r} = \frac{10}{21}$$

$$\frac{\dfrac{10!}{(10-r-2)!\,(r+2)!}}{\dfrac{10!}{(10-r)!\,.\,r!}} = \frac{10}{21}$$

$$\frac{(10-r)!\,.\,r!}{(8-r)!\,.\,(r+2)!} = \frac{10}{21}$$

$$\frac{(10-r)(9-r)(8-r)!}{(8-r)!}\ \frac{r!}{(r+2)(r+1)\,r!} = \frac{10}{21}$$

$$\frac{(10-r)(9-r)}{(r+2)(r+1)} = \frac{10}{21}$$

$$\frac{90-10r-9r+r^2}{r^2+2r+r+2} = \frac{10}{21}$$

$$\frac{90-19r+r^2}{r^2+3r+2} = \frac{10}{21}$$

$$1890-399r+21r^2 = 10r^2+30r+20$$

$$11r^2-429r+1870 = 0$$

$$r^2-39r+170 = 0$$

$$(r-34)(r-5) = 0$$

$r \neq 34$ or $r = 5$

09. $\;{}^nC_{r-1} : {}^nC_r : {}^nC_{r+1}$

$= 20 : 35 : 42$, *find n & r*

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{20}{35}$$

$$\frac{\dfrac{\cancel{n!}}{(n-r+1)!\,.\,(r-1)!}}{\dfrac{\cancel{n!}}{(n-r)!.r!}} = \frac{20}{35}$$

$$\frac{(n-r)!\,.\,r!}{(n-r+1)!\,.\,(r-1)!} = \frac{20}{35}$$

$$\frac{(n-r)!}{(n-r+1)!}\,.\,\frac{r!}{(r-1)!} = \frac{20}{35}$$

$$\frac{\cancel{(n-r)!}}{(n-r+1).\cancel{(n-r)!}}\;\frac{r\,.\,\cancel{(r-1)!}}{\cancel{(r-1)!}} = \frac{20}{35}$$

$$\frac{r}{n-r+1} = \frac{4}{7}$$

$$7r = 4n - 4r + 4$$

$$11r - 4n = 4 \quad \dots\dots\dots \quad (1)$$

Now

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{35}{42}$$

$$\frac{\dfrac{n!}{(n-r)!\,.\,r!}}{\dfrac{n!}{(n-r-1)!.(r+1)!}} = \frac{35}{42}$$

$$\frac{(n-r-1)!\,.\,(r+1)!}{(n-r)!\,.\,r!} = \frac{35}{42}$$

$$\frac{(n-r-1)!}{(n-r)!\,.}\;\frac{(r+1)!}{r!} = \frac{35}{42}$$

$$\frac{\cancel{(n-r-1)!}}{(n-r).\cancel{(n-r-1)!}}\;\frac{(r+1).\cancel{r!}}{\cancel{r!}} = \frac{35}{42}$$

$$\frac{r+1}{n-r} = \frac{5}{6}$$

$$6r + 6 = 5n - 5r$$

$$11r - 5n = -6 \quad \dots\dots\dots \quad (2)$$

Solving (1) & (2)

$$\begin{array}{rcr} 11r - 4n & = & 4 \\ 11r - 5n & = & -6 \\ - \quad\; + & & + \\ \hline n & = & 10 \end{array}$$

subs in 1 : $11r - 40 = 4$

$11r = 44$

$r = 4$

10. *${}^{n}C_{r-1} = 495$; ${}^{n}C_{r} = 220$; ${}^{n}C_{r+1} = 66$, find n & r*

$$\frac{{}^{n}C_{r-1}}{{}^{n}C_{r}} = \frac{495}{220}$$

$$\frac{\dfrac{\cancel{n!}}{(n-r+1)!\,.\,(r-1)!}}{\dfrac{\cancel{n!}}{(n-r)!.r!}} = \frac{9}{4}$$

$$\frac{(n-r)!\,.\,r!}{(n-r+1)!\,.\,(r-1)!} = \frac{9}{4}$$

$$\frac{(n-r)!}{(n-r+1)!}\,.\,\frac{r!}{(r-1)!} = \frac{9}{4}$$

$$\frac{\cancel{(n-r)!}}{(n-r+1).\cancel{(n-r)!}}\ \frac{r\,.\,\cancel{(r-1)!}}{\cancel{(r-1)!}} = \frac{9}{4}$$

$$\frac{r}{n-r+1} = \frac{9}{4}$$

$4r = 9n - 9r + 9$

$13r - 9n = 9$ ……… (1)

Now

$$\frac{{}^{n}C_{r}}{{}^{n}C_{r+1}} = \frac{35}{42}$$

$$\frac{\dfrac{n!}{(n-r)!\,.\,r!}}{\dfrac{n!}{(n-r-1)!.(r+1)!}} = \frac{220}{66}$$

$$\frac{(n-r-1)!\,.\,(r+1)!}{(n-r)!\,.\,r!} = \frac{10}{3}$$

$$\frac{(n-r-1)!}{(n-r)!\,.}\ \frac{(r+1)!}{r!} = \frac{10}{3}$$

$$\frac{\cancel{(n-r-1)!}}{(n-r).\cancel{(n-r-1)!}}\ \frac{(r+1).\cancel{r!}}{\cancel{r!}} = \frac{10}{3}$$

$$\frac{r+1}{n-r} = \frac{10}{3}$$

$3r + 3 = 10n - 10r$

$13r - 10n = -3$ ……… (2)

Solving (1) & (2)

$$\begin{array}{rcl} 13r - 9n & = & 9 \\ 13r - 10n & = & -3 \\ - \quad\ + & & + \\ \hline n & = & 12 \quad , r = 9 \end{array}$$

11. *${}^{14}C_{5} + {}^{14}C_{6} + {}^{15}C_{7} + {}^{16}C_{8} = {}^{17}C_{x}$,* find x

Using $\underline{{}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}}$

$\underline{{}^{14}C_{5} + {}^{14}C_{6}} + {}^{15}C_{7} + {}^{16}C_{8} = {}^{17}C_{x}$,

$\underline{{}^{15}C_{6} + {}^{15}C_{7}} + {}^{16}C_{8} = {}^{17}C_{x}$,

$\underline{{}^{16}C_{7} + {}^{16}C_{8}} = {}^{17}C_{x}$,

${}^{17}C_{8} = {}^{17}C_{x}$,

${}^{17}C_{8} = {}^{17}C_{9} = {}^{17}C_{x}$ $\therefore$ x = 8 OR 9

12. ${}^{25}C_{4} + {}^{25}C_{5} + {}^{26}C_{6} + {}^{27}C_{7} = {}^{28}C_{r}$,

find r

Using $\underline{{}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}}$

$\underline{{}^{25}C_{4} + {}^{25}C_{5}} + {}^{26}C_{6} + {}^{27}C_{7} = {}^{28}C_{r}$,

$\underline{{}^{26}C_{5} + {}^{26}C_{6}} + {}^{27}C_{7} = {}^{28}C_{x}$,

$\underline{{}^{27}C_{6} + {}^{27}C_{7}} = {}^{28}C_{x}$,

${}^{28}C_{7} = {}^{28}C_{x}$,

${}^{28}C_{7} = {}^{28}C_{21} = {}^{28}C_{x}$ $\therefore$ x = 7 OR 21

13. $^{12}C_5 + 2.^{12}C_4 + {}^{12}C_3 = {}^{14}C_x$,

find x

$$\underbrace{^{12}C_5 + {}^{12}C_4}_{\downarrow} + \underbrace{^{12}C_4 + {}^{12}C_3}_{\downarrow} = {}^{14}C_x$$

$$^{13}C_5 + {}^{13}C_4 = {}^{14}C_x$$

$$^{14}C_5 = {}^{14}C_x$$

Now ; $^{14}C_5 = {}^{14}C_9 = {}^{14}C_x$

$x = 5 \text{ OR } 9$

14. $^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3.$

$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + \underline{^{47}C_3 + {}^{47}C_4}$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + \underline{^{48}C_3 + {}^{48}C_4}$

$= {}^{51}C_3 + {}^{50}C_3 + \underline{^{49}C_3 + {}^{49}C_4}$

$= {}^{51}C_3 + \underline{^{50}C_3 + {}^{50}C_4}$

$= \underline{^{51}C_3 + {}^{51}C_4}$

$= {}^{52}C_4$

Hence ;

$$^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3. = {}^{52}C_4$$

15. $^{20}C_{2n} = {}^{20}C_{n^2-4}$ find n

$^{20}C_{2n} = {}^{20}C_{20-2n} = {}^{20}C_{n^2-4}$

$20 - 2n = n^2 - 4$

$n^2 + 2n - 24 = 0$

$(n + 6)(n - 4) = 0$

$n \neq 6 \;;\; n = 4$

16. $^{18}C_{2r} = {}^{18}C_{r^2+3}$ find r

$^{18}C_{2r} = {}^{18}C_{18-2r} = {}^{18}C_{r^2+3}$

$18 - 2r = r^2 + 3$

$r^2 + 2r - 15 = 0$

$(r + 5)(r - 3) = 0$

$r \neq 5 \;;\; r = 3$